

# APOCALYPSE SOON <sup>†</sup>

by

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## ABSTRACT

Based upon a simple vacuum Lagrangian, comprising cosmological and quadratic scalar field terms, a cosmological model is presented whose history is indistinguishable from that of an innocuous low-density cold dark matter (CDM) universe, but whose future is very much shorter. For sensible values of the deceleration parameter ( $0 < q_0 < 1$ ), its age is greater than 85% of the Hubble time, thus resolving the current version of the age crisis, which appears to be that  $t_0 \sim 1/H_0$  while  $q_0$  is significantly positive.

**Key words:** cosmology – observations – theory – dark matter.

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## 1 INTRODUCTION

That there is something wrong with our understanding of the dynamics of large astronomical systems, characterised by length scales ranging from 10 kpc (galaxies) to 100 Mpc (deviations from the Hubble flow) seems to be beyond doubt, which problems have been ‘solved’ by the introduction of dark matter. Beyond 100 Mpc, there are indications that the Hubble expansion is slowing down, (Perlmutter et al. 1996, 1997a,b), to an extent which would also require significant amounts of dark matter, corresponding to a deceleration parameter  $q_0 \sim 1/2$ .

I have long had an interest in the idea that the solution to the mystery of extra-galactic dynamics might be found in the very fabric of space-time (i.e. the vacuum), rather than its contents (Jackson 1970, 1992; Jackson & Dodgson 1996, 1997). Traditionally the concept of vacuum energy finds its expression in the form of a cosmological term in Einstein’s equations, which in effect assigns an energy-density  $\lambda/8\pi G$  to the vacuum. We have argued (Jackson & Dodgson 1998) that the natural generalization of this concept introduces in addition a scalar field  $\phi$ , with corresponding Lagrangian  $\mathcal{L} = \dot{\phi}^2/2 - V(\phi)$ , where the potential  $V(\phi)$  has a minimum at  $\phi = 0$ . The energy-density attributed to the vacuum is now  $\dot{\phi}^2/2 + V(\phi)$ , and in the proposed model  $V(\phi)$  has the simple form  $\lambda/8\pi G + \omega_c^2 \phi^2/2$ , which is generic in the sense that it is not tied to a particular field-theoretic model, but is just a Maclaurin expansion in which only the first three terms are retained;  $\omega_c$  is the Compton frequency of an associated ultra-light boson (cf. Frieman et al. 1995). Thus

$$\rho_{VAC} = \dot{\phi}^2/2 + \lambda/8\pi G + \omega_c^2 \phi^2/2. \quad (1)$$

This is of course the language of chaotic inflationary cosmology (Linde 1983), but the point here is that with the appropriate value of  $\omega_c$  (i.e.  $\omega_c^{-1} \gtrsim$  current Hubble time, rather than a tiny fraction of a second), ‘inflation’ is something which might have happened in the recent past, rather than a long time ago (Frieman et al. 1995; Jackson & Dodgson 1998). In addition to  $\rho_{VAC}$  a real Universe would require a trace of more conventional matter, Cold Dark Matter plus baryonic, or possibly just the observed baryons; there would be no technical difficulties associated with such a component, but in what follows I shall neglect its marginal dynamical effects, the inclusion of which would serve only to obscure the main points of this communication. The general view adopted here is that ordinary matter is just flotsam floating on a deep and possibly rough scalar sea.

Our initial motivation in this context (Jackson & Dodgson 1998) was occasioned by the recent upward trend in measures of  $H_0$  (Pierce et al. 1994; Freedman et al. 1994; Tanvir et al. 1995), when the observational evidence seemed to indicate that  $t_0 > 1/H_0$ , and that  $q_0 > 0$ , which behaviour is allowed by the class of models described above; the scalar field mimics a positive cosmological constant in an inflationary slow-roll phase until the Hubble time exceeds  $\omega_c^{-1}$ , after which deceleration commences. The upward trend in  $H_0$  values has been reversed somewhat by data from the Hipparcos astrometry satellite (Feast & Catchpole 1997; see, however, Madore & Freedman 1998), and the age crisis may not be as severe as it seemed a few years ago; the current view is probably that  $t_0 \sim 1/H_0$ , corresponding for example to a low-density CDM model, but significantly longer than the value  $2/3 \times 1/H_0$  demanded by the canonical flat CDM model. However, quite apart from these considerations, I believe that the class of models described by equation (1) deserves further study, and I present here one which seems to me to have some particularly interesting properties.

## 2 STAGFLATION

A peculiar feature of the class is that it offers two mechanisms for generating a cosmological con-

stant, the necessarily positive slow-roll ( $\dot{\phi} \ll \omega_c \phi$ ) inflationary term  $4\pi G \omega_c^2 \phi^2$ , and the conventional  $\lambda = 8\pi G V(0)$  deriving from a non-zero minimum value of  $V$ . There seems to be little point in retaining the conventional  $\lambda$  term as an extra positive contribution, but interesting variations in behaviour arise if  $V(0)$ , and hence  $\lambda$ , is negative; a possibility which has particularly captured my imagination is a slow-roll expanding phase in which  $\phi = \phi_0$  initially and the last two terms in equation (1) cancel:

$$\lambda = -4\pi G \omega_c^2 \phi_0^2, \quad (2)$$

which might be called stagflation.

The scalar field has effective density  $\rho_\phi = (\dot{\phi}^2 + \omega_c^2 \phi^2)/2$  and pressure  $p_\phi = (\dot{\phi}^2 - \omega_c^2 \phi^2)/2$ , and for a model in which these are dominant, the Friedmann equations for the scale factor  $R(t)$  are

$$\ddot{R} = -\frac{4\pi}{3} G(\rho_\phi + 3p_\phi)R + \frac{1}{3} \lambda R = -\frac{4\pi}{3} G(2\dot{\phi}^2 - \omega_c^2 \phi^2)R + \frac{1}{3} \lambda R, \quad (3)$$

$$\dot{R}^2 = \frac{8\pi}{3} G \rho_\phi R^2 + \frac{1}{3} \lambda R^2 - kc^2. \quad (4)$$

The scalar field is governed by the equation

$$\ddot{\phi} + 3H\dot{\phi} + \omega_c^2 \phi = 0, \quad (5)$$

where  $H = \dot{R}/R$  is Hubble's 'constant' (see for example Peebles 1993).

During the slow-roll phase we have  $\dot{\phi} \ll \omega_c \phi$ , and thus with  $\lambda = -4\pi G \omega_c^2 \phi_0^2$  equation (3) gives  $\ddot{R} \sim 0$ , which brings us to the first interesting feature of the model:

i) its history is indistinguishable from that of an innocuous low-density CDM universe, but its future might be very different.

It is possible to trace analytically the beginnings of said future, which is increasingly dominated by an imbalance as  $\phi$  drifts away from the value indicated by equation (2). During the slow-roll phase, we have  $H = 1/t$ , and equation (5) becomes

$$\ddot{\phi} + \frac{3}{t}\dot{\phi} + \omega_c^2 \phi = 0. \quad (6)$$

An exponential inflationary phase is characterised by a constant value of  $H$ , when the  $t$  in the second term of equation (6) has a fixed value  $t_H$ , and  $\ddot{\phi} \ll 3\dot{\phi}/t_H$ , so that the first term in equation (6) can be neglected. This is not the case here, but a non-singular series solution for  $\phi$  is easily developed, to give

$$\phi = \phi_0(1 - \omega_c^2 t^2/8 + \omega_c^4 t^4/192 - \dots), \quad (7)$$

To order  $\omega_c^2 t^2$ , the scalar density and pressure are now

$$\rho_\phi = \frac{\omega_c^2 \phi_0^2}{2} \left(1 - \frac{3}{16} \omega_c^2 t^2\right), \quad p_\phi = -\frac{\omega_c^2 \phi_0^2}{2} \left(1 - \frac{5}{16} \omega_c^2 t^2\right), \quad (8)$$

and equation (3) becomes

$$\ddot{R} = -\frac{1}{8}(4\pi G \omega_c^2 \phi_0^2) \omega_c^2 t^2 R = -\frac{1}{8} |\lambda| \omega_c^2 t^2 R, \quad (9)$$

or, in terms of the deceleration parameter  $q = -\ddot{R}R/\dot{R}^2$ ,

$$q = \frac{1}{8}|\lambda|\omega_c^2 t^4 \Rightarrow t(q) = \left(\frac{8q}{|\lambda|\omega_c^2}\right)^{1/4}. \quad (10)$$

Thus it turns out that, for fixed  $q_0$  and  $t_0$ ,  $|\lambda|$  and hence  $\phi_0$  can be arbitrarily large, as long as  $\omega_c$  is appropriately small:

$$\omega_c = \left(\frac{8q_0}{|\lambda|t_0^4}\right)^{1/2}. \quad (11)$$

However, to allow an exit from the slow-roll phase, and the possibility that  $q_0 \neq 0$ ,  $\omega_c$  cannot be exactly zero. The second attractive feature of the model is thus:

ii) it serves as a paradigm for what might be happening in the real vacuum, in which the huge zero-point energy suggested by quantum field theory is renormalized by a counter term.

To this order of approximation, equation (9) integrates to give the scale factor explicitly as

$$R(t) = t \left(1 - \frac{|\lambda|\omega_c^2 t^4}{160}\right). \quad (12)$$

It is clear that evolution is determined by the composite parameter  $|\lambda|\omega_c^2$ , and that only this combination can be fixed by cosmological observations, for example via equation (10) from knowledge of  $q_0$  and  $t_0$ . Equation(4) gives  $\dot{R}^2 = -kc^2$  initially, so that  $k = -1$  and the model is an open one.

To follow the evolution in full, equations (3) to (5) must be integrated numerically, which is best achieved by using the above analytical solution to give a convenient starting point which avoids the initial singularity. Figure 1 shows the resulting trajectory, normalized in terms of both  $R$  and  $t$  to the point where  $q_0 = 1/2$ , which curve is universal in the sense that it does not depend upon the particular value of  $|\lambda|\omega_c^2$ , in the limit of large values of this parameter. Figure 1 illustrates a third feature of this universe:

iii) for all conceivable values of the deceleration parameter, its age is greater than 70% of the Hubble time,

which aspect, from the point of practical cosmology, is probably its most important. This behaviour is quantified in Figure 2, which shows  $Ht$  as a function of  $q$ , together with the conventional CDM curve for comparison; thus for example  $q_0 = 1/2$  corresponds to  $H_0 t_0 = 0.91$  here, much greater than the CDM figure of 0.67. There is however a terrible price to pay for an extended history; violent re-collapse is inevitable, and the future is very much shorter than might be expected; the final stages of the collapse are in fact dominated by the  $\dot{\phi}^2$  terms in the expressions for  $\rho_\phi$  and  $p_\phi$ , when  $p_\phi \sim +\rho_\phi$ , and the scalar terms in equation (3) reinforce the effects of the negative cosmological constant. The title of this note is explained by such behaviour.

In writing this letter I am well aware of the possibility that a reworking of stellar evolution theory, coupled with the recent downturn in measures of  $H_0$ , may re-establish a more conventional view of cosmic history. A putative cosmological model must expect to run the gauntlet of an increasing number of neo-classical observational tests ( $z \lesssim 4$ ), and of modern tests based upon structure formation and the cosmic microwave background ( $z \lesssim 1000$ ), but in the context of this model I would not be prepared to further action until such time as the age crisis has a firmer observational basis. The crisis may take the extreme form  $t_0 > 1/H_0$ ,  $q_0 > 0$  discussed by Jackson & Dodgson (1998), or the

more modest form discussed here, namely  $t_0 \lesssim 1/H_0$  but too long to be compatible with significant deceleration. Radical measures would seem to be unavoidable, if either possibility is confirmed.

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## FIGURE CAPTIONS

Figure 1. Scale factor  $R(t)$  for the stagflationary model; the asterisk marks the point at which  $q = 0.5$ .

Figure 2. The continuous line shows the age of the stagflationary model in Hubble units, as a function of the deceleration parameter  $q$ ; the dashed line is the corresponding CDM curve. The dotted line shows the time remaining in the stagflationary case; the corresponding CDM times are too long to appear here, being  $Ht \geq 1.27$  for  $q \leq 3$ , and of course infinite for  $q \leq 1/2$ .

Figure 1

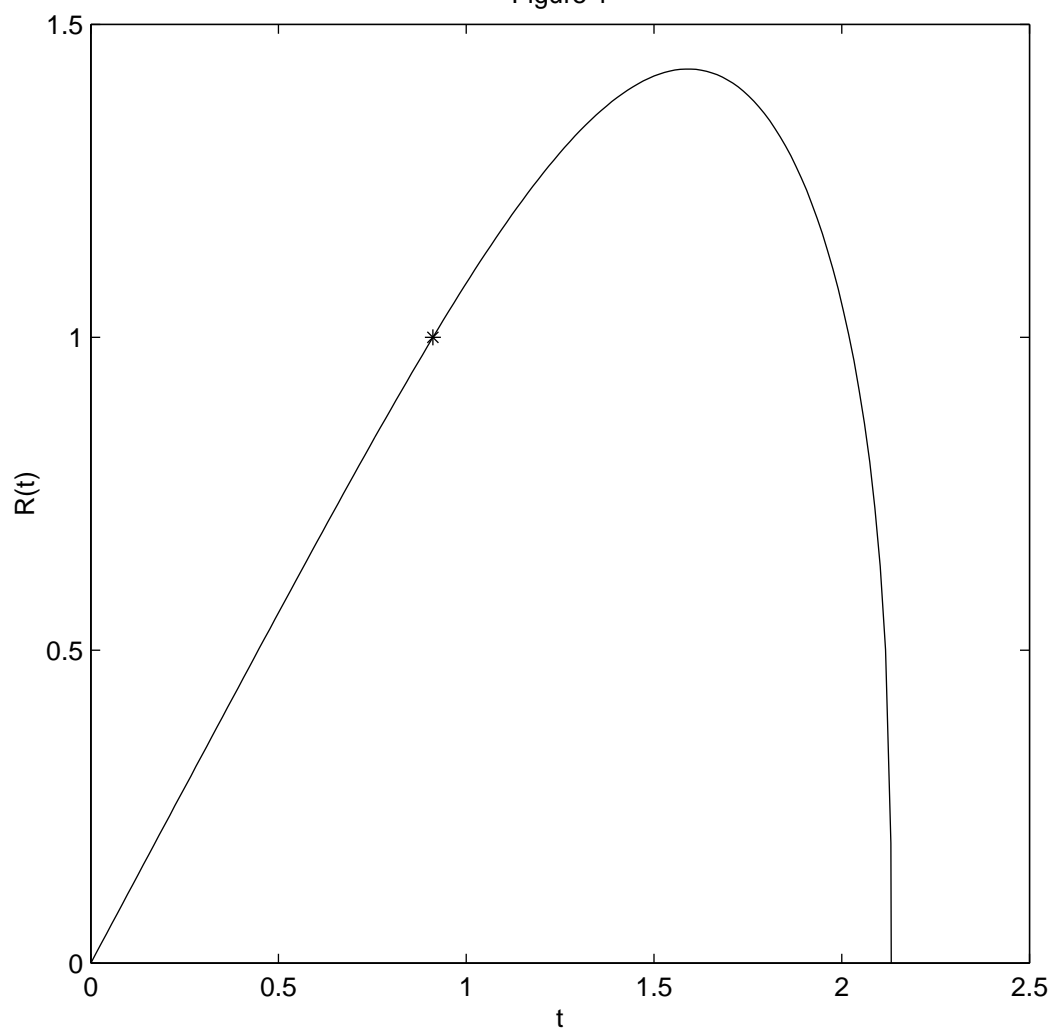


Figure 2

